### Definition: Cardinality

Let M be a set. We define the cardinality of a set asM : = n, ifn∞∈, if ℕ∪M0 has infinitely many elements.is the number of elements in M is

∞ is the mathematical symbol for infinity.

3.1.2. and let | = 4M'2.⊆ M2 with M'2 := {m ∈ M2|m is even}

and we call it Please note that the empty set Let always included in every power set.M be a set. Then we define the power set of P(M). ∅ is always a possible subset of any other set! Thus M as the set of all possible subsets of ∅ isM

2.1. Please note that the empty set is always an element of the power set of a set! TheLet Let Pc, dMMM21P := {1,2,3} := {a,b,c,d}M2 = ∅. Then, 1 , 2 , 3a, c, d,, 1a, , 2b,,,b, c, d1a, c, 3 ,, ,2a,, a, b, c, d d3 ,, 1b, c, 2, 3, b, d,

power set of a finite set is always finite, the power set of an infinite set is infinite.

Let Theorem: Cardinality of the Power SetProof:M be a finite set with |M| = n ∈ ℕ0. Then |P(M)| = 2n.

Base case: If We prove the statement of this theorem with induction:induction base case is valid.element, namely the empty set itself. So it follows that n = 0, then M is the empty set. The power set of the empty set has only one|P(M)| = 1= 20 and thus the Induction step: Now let with ments. We must show that from this it follows that n + 1 elements. n ≥ 0. Let |P(M)| = 2n be valid for each finite set |P(M')| = 2n+1 for every finite set M with n ele-M' differentLet M' be a set with subsets M'n + 1 has. Let elements, thus L' be any subset of M' := {aM'1, i.e., , ., aL' ⊆ M'(n+1)}. We investigate how many. Then there are exactly

1.2. If Possibility: aan+1 ∈ L' M , ., a|P(M)|} two possibilities:

1:= 2 . Since different is of the form , there are therefore subsets of L' it follows that 2 can exist. Thus1 n . WeM'

possibilities of how subsets of

entfor 2:and If L' subsets and it follows 2 subsets of L'M.2, then n, because subsets of M' that do not contain L' is a subset of |P(M')| = 2|P(M)| = 2M', which do not contain M := {an. Consequently, there are . a□ . M'1, thus has a total of ., aann+1}2. We know that there are . subsets of 2n differentM'2 + 2 that contain possibilities = 2 ⋅ 22ndiffer-a = Since there are no other possibilities, there are exactly 2n+1 n n+1 n+1 n n n nn+1